What Makes Time Special

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What is the difference between time and space? This question, once a central one in metaphysics, has not been treated kindly by recent history. By joining together space and time into spacetime Minkowski sapped some of the spirit out of this project. That is unfortunate, however, for even in relativistic theories there remain sharp and important metrical and topological distinctions between the timelike and spacelike directions of spacetime. Questions about what these differences are, why they exist and how they are related are fascinating. Why, for instance, is time one-dimensional in virtually all physical theories? What does the “minus sign” in the relativistic metric have to do with time? Is there a connection between the two? At a time when researchers in quantum gravity regularly propose speculative theories with no time at all, a better understanding of time in physics is all the more important—even if only to see what is lost by its absence.

This paper proposes a novel answer to the question of what distinguishes time from space: the temporal direction is that direction on the manifold of events in which our best theories can tell the strongest, most informative “stories.” Put another way, time is that direction in which our theories can obtain as much determinism as possible. Time is not only the "great simplifier" [1], but it is also the great informer. I make two arguments. The first is a general one based on an empiricist conception of law of nature defending the idea that informativeness helps determines what is temporal. The second is a more specific illustration of the first: understanding informative ‘strength’ as having a well-posed Cauchy problem, I show that for a wide class of equations, the desire for strength does indeed distinguish the temporal direction. Not only that, but the direction of strength shows how the topological and metrical features special to time are related to one another.

1. TIME IN PHYSICS

Before getting to our project, we first need to recall the ways in which time is distinguished from space in contemporary physics. Since very little is invariant under a change of the spatial and temporal directions, one could answer this question by simply listing the equations non-invariant under a transformation of timelike and spacelike directions. But this wouldn't be very illuminating. Instead, let's begin by examining those properties commonly attributed to time but not space, and also, those features that do not vary with initial conditions. Thus we'll ignore features like being Hausdorf, connected, and so on, since they are also features associated with being spatial. And we'll also bracket the so-called direction of time, for that is typically considered a feature of the (thermodynamic, etc.) processes in time rather than of time itself. Reflection on physics reveals three features of time possibly surviving the above winnowing process: the one-dimensionality of time, the metrical difference between time and space, and the lack of what we might call 'free mobility'.

First, the dimensionality. Every successful physical theory purporting to be fundamental has judged time to be one-dimensional. In classical physics, assuming that the instants of time form a continuum under the 'earlier than or simultaneous with' relation and that this order relation determines the open sets that form a basis for this topological structure, the set of instants is topologically one-dimensional. The set of instants is described by $\mathbf{R}^1$ and the whole set of events is given by $\mathbf{R}^3 \mathrm{x} \mathbf{R}^1$. In relativistic physics, however, there isn't *the* set of instants. One can't just grab a set of events corresponding to the 'temporal' ones and check its dimensionality. Still, time is one-dimensional in relativistic physics. Consider a point *p* on a timelike curve and a four-velocity field $\mathbf{v}^a$. Consider a vector $\mathbf{w}^a$ at *p*. Then $\mathbf{w}^a$ can be decomposed into components parallel to and orthogonal to $\mathbf{v}^a$. The set of orthogonal vectors form a three-dimensional subspace in the tangent space M_p at *p*. The set of parallel vectors form a one-dimensional subspace in M_p. Interpreting the first set as spatial and the second as temporal, we then have the sense in which the set of timelike directions is one-dimensional. In effect, we have sliced up spacetime into space and time on an infinitesimal simultaneity slice.

Second, metrical difference. Because it determines the causal structure, the metric is arguably the most central feature of spacetime. It is therefore highly significant that the metric, whether relativistic or classical, distinguishes time from space. In classical physics the distinction comes in the form of separate metrics. There is a (degenerate) metric for space *h* and a (degenerate) metric for time *t*. In relativistic physics, there is instead one metric *g*, but it distinguishes the spatial and temporal components by a famous minus sign. The Minkowski interval, for instance, is

$$g = ds^2 = (dx_1)^2 + (dx_2)^2 + (dx_3)^2 - (dx_4)^2$$

where in this convention x_4 is considered time. Or put in a coordinate-invariant way (assuming the manifold is connected), the difference between spacelike and timelike directions appears through the signature of the metric.[1]

Third, free mobility. One of the most obvious differences between time and space is that we have relatively free mobility in the spatial directions but not the temporal ones. Our worldlines can bend back and forth in the spatial directions but not the temporal ones. Although time travel may be physically possible in some sense, so far it has been noticeably off-limits, and prima facie, this represents an interesting difference between the worldliness in the spacelike and timelike directions.

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[1] Given a spacetime metric *g*, we can find an orthonormal basis $\mathbf{v}_1 \ldots \mathbf{v}_n$ of the tangent space at each point *p* of *M*. This is a basis such that $g(\mathbf{v}_\mu, \mathbf{v}_\nu)=0$ if $\mu \neq \nu$ and $g(\mathbf{v}_\mu, \mathbf{v}_\nu)=\pm 1$ if $\mu=\nu$. Let the number of basis vectors with $g(\mathbf{v}_\mu, \mathbf{v}_\mu)=+1$ be *p* and the number of basis vectors with $g(\mathbf{v}_\mu, \mathbf{v}_\mu)=-1$ be *q*. Then the metric has signature (*p*,*q*). In relativity we assume the metric is nondegenerate, so $p+q=d$, where *d* is the dimensionality of the spacetime. If the signature of *g* is (*d*-1,1), then the metric is called *Lorentzian*. Since the "time" component is given by the vector **v** such that $g(\mathbf{v}_\mu, \mathbf{v}_\mu)=-1$, time is distinguished from space in virtue of the metric being Lorentzian.

2. LAWS AND TIME

The topic of laws of nature is one fraught with controversy. Despite this, it is interesting that the best empiricist account of laws available suggests two very deep features of time. The first feature is that time is the so-called "great simplifier". The second and more novel feature is that time is also the great informer.

Theories of laws of nature come in many forms, but I've always favored versions with an empiricist slant. Empiricist theories seek to explain the laws given the distribution of actual or observed facts, rather than going the other way round and explaining why the facts are what they are in virtue of the laws. The most attractive such theory is arguably the “Best System” theory of laws (see, e.g., [2] and references therein).

A rough sketch of the theory is as follows. Consider various deductive systems, each of which makes only true claims about what exists. Some of these theories will be very simple, others will be very informative. Now run a competition among these systems looking for the one that *best balances* simplicity and strength. Simplicity is measured with respect to a language that contains a primitive predicate for each fundamental property, e.g., basic field values. Strength is informativeness about matters of particular fact. The laws of nature are the axioms that all the “best systems” have in common. The motivation for the theory is the idea that physical laws seek to describe accurately as much of the world as possible in a compact way.

Now turn to time. Time, it is often said, is the “great simplifier”. What this means is that the temporal metric is chosen to make motion look simple. According to Poincaré [3] and others, we pick a measure of duration that yields the most powerful and simple physics. This basic idea about time is implied by the Best System theory of lawhood.

This fact hardly distinguishes time. Space is also the great simplifier. What goes for clocks and temporal duration goes for meter sticks and spatial duration. And in relativistic theories spacetime intervals are also so defined. For this reason simplicity alone will not distinguish time from space.

However, informativeness may well distinguish time from space. In balancing simplicity with strength, a best system will not include a random catalogue of everything that happens. It will instead contain a way to generate some pieces of the domain of events given other pieces. In other words, it will favor algorithms, and short ones at that. The more of what happens that is generated by small input the better. Further, it might be that the distribution of basic properties on the manifold picks out one set of directions as special in this regard.

To help fix ideas, consider the actual distribution of matter in the universe at a very large and coarse-grained scale, like that of interest to modern cosmology. Looking at this distribution, it’s a remarkable fact that the universe is approximately isotropic and homogeneous in three of its four directions (relative to co-moving coordinates). The universe looks the same no matter where you are and what direction you look. However, it isn't isotropic and homogeneous in all four directions. If it were, then at these scales the best system probably could not single out one direction over any other. In a world like ours, however, one can write simple but strong laws, e.g., for the line element for a FRW universe, for the evolution of the matter density in such a universe, with reference to that special direction. I don't wish to make too much of this example, but it does provide a quick illustration of how considerations of strength and simplicity might "find" time.

3. TIME IS THE GREAT INFORMER

The above considerations motivate the following proposal. Time is that direction in spacetime in which we can tell the strongest or most informative stories. To phrase it slightly more carefully, suppose we have a manifold M^d endowed with a metric g. The points of M are interpreted as elementary point events and g gives the distances between any pair of these events. In none of this do we presuppose a prior picking out of the temporal directions. Then the very general *Proposal 1* is:

> A temporal direction at a point p on $\langle M,g \rangle$ is that direction in which our best theory tells the strongest, i.e., most informative, "story."

The set of all such directions is then the temporal direction. So that we don't fall irretrievably into terminological muddles, please remember that the metric g, if indefinite, may well designate some directions "timelike" and some "spacelike", but there is no reason—so far as Proposal 1 is concerned—that the most informative "temporal" direction must be the "timelike" (i.e., $g(\mathbf{v},\mathbf{v})<0$) direction. A priori, we do not expect the direction of strength to always line up with the direction called "timelike" by indefinite metrics. A connection between the two will only appear later.

Let's now think about strength. If an algorithm, given some input, could get back *everything* that happens, that would be best. A deterministic theory is maximally strong in that respect. Another way of thinking about Proposal 1, therefore, is that time is that direction of spacetime in which the best system can get the most determinism. Call a history H a map from $\mathbf{R}$ to tuples of the fundamental properties, where for any t in R, $H(t)$ gives the state of the fundamental properties at t. Then a theory is deterministic iff for any pair of histories, H_1, H_2, that satisfy the laws of physics, if $H_1(t) = H_2(t)$ at one time t, then $H_1(t) = H_2(t)$ for all t [2]. Note that this definition presupposes a time versus space split; in fact, it presupposes that we have a global time function.[2] However, if our spacetime meets the conditions necessary to define a global time function, then we can easily turn this around and define time in terms of determinism: a smooth map $t: M \rightarrow R$ is a global time function if it's true that for histories that satisfy the laws of physics, if any pair agree at one value of t then they agree for all values of t.[3]

In other cases we may want strength to distinguish a timelike direction even where no global time function is definable. For example, there exist spacetimes with closed timelike curves (and hence no global time function) that nonetheless are perfectly deterministic [4]. And there exist other spacetimes that cannot be foliated via everywhere spacelike hypersurfaces, e.g., Gödel spacetime, yet where one may locally want to distinguish time. In these cases one may still distinguish space from time at a point.

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[2] A global time function is a smooth map $t: M \rightarrow R$ such that for any $p,q \in M$, $t(p) < t(q)$ iff there is a future directed timelike curve from p to q.

[3] Because "bad" choices of foliation can ruin determinism, some care is needed; see [2] for the necessary modifications.

Let me emphasize that my proposal in no way commits me to asserting determinism true in our world. Indeterministic theories are strong too. Most stochastic processes in physics are Markovian. A process $q(t)$ is Markovian if (say) $q(t_1)$ alone predicts $q(t_1+dt)$, where no previous values $q(t_n)$, $t_n < t_1$, are needed. Knowing something about the future and being able to screen off the past is certainly a kind of informative strength. Hence, a natural probabilistic analog of my claim is that time is that dimension in which the laws are Markovian.[4]

Even at this abstract level I believe this proposal is quite attractive. First, it has a number of virtues compared to its rivals. Since it is an empiricist theory, we have an understanding of why the laws pick out time. The difference between time and space is not found simply in a metaphysical primitive one has and the other doesn't. The difference ultimately lay in the distribution of fundamental physical properties.

Second, the proposal explains various features of time that are otherwise mysterious. For instance, why is time typically one-dimensional in physical theories? The Best System helps explain this tendency, for one can identify theoretical pressure from systemization for time to be one-dimensional. Suppose for simplicity that our fundamental theory is deterministic. Then what incentive is there for the best system to get determinism again in another direction? The theory is already maximally strong. It's hard to imagine that one could even find two such directions, especially in a complex world like ours. And it is even harder to imagine this happening without a compensating loss of simplicity. Informativeness in one set of directions is enough.

Third, the theory fits well with the history of science. Reflecting on our physical theories, we have never come close to having a fundamental theory that is deterministic in what we call a spatial direction. From Newtonian mechanics to the standard model, the direction we would pick out as the direction of strength meshes with what is the temporal direction under the theory's intended interpretation.

4. TIME, INFORMATION AND PDE'S

When Proposal 1 climbs down from the lofty heights of philosophy it looks even more attractive. Let's explore the links between aspects of time that can be forged when we make, for the sake of illustration, some detailed commitments about informativeness and the types of physics in the world.

Focusing on theories that are maximally strong *for us*, we can desire something better than determinism, namely, a well-posed Cauchy problem. A partial differential equation defined over a certain domain, possibly supplemented by boundary conditions, is well-posed if (1) there is a solution u for *any* choice of the data D, where D belongs to an admissible set X, (2) the solution u is uniquely determined within some set Y by the data D, and (3) the solution u depends "continuously" on the data D, according to some

[4] Let me also stress that being deterministic or Markovian are merely *marks* of strength. Confining attention to the marks of strength and not strength itself would be a mistake. The degree to which a theory is informative is determined by how much of the actual world it manages to imply, not (in the general theory, at least) by formal characteristics.

suitable topology. To make sense, condition 3 needs a precise characterization of topology involved. If induced by a norm $\|\ .\ \|$, 3 implies that there is a nondecreasing nonnegative function $F(x)$ such that $\|u(x)\| \leq F(x)\|u(0)\|$, $x>0$, for any solution $u(x)$. Notice that although we usually think of marching Cauchy data forward in time, the concept of well-posed Cauchy problem is entirely free of temporal presuppositions.

The reasons why we want existence and uniqueness of solution are clear: our theory is most informative if it tells us exactly what will happen. If we understand strength in terms of strength *for us*, it is equally clear why we would want the solution to continuously depend on the data. In that case, the solution $u(t)$ is equal to some continuous function G of the initial data, i.e., $u(t)=G(u(0),u_t(0))$. Suppose one needs to find the position of some asteroid at a particular time t within accuracy $\varepsilon>0$. In real life we measure the initial data with a certain error, which we can often make arbitrarily small by investing enough energy and time. However, energy and time are in demand, so we narrow down the initial data only so far. Thanks to the continuity of G, we're at least guaranteed that there are positive numbers d_1 and d_2 such that if $|u(0)-u'(0)| < d_1$ and $|u_t(0)-u_{t'}(0)| < d_2$, then $|u(t)-u'(t)| < \varepsilon$. If, by contrast, the solution depends on the data in a discontinuous way, then that will mean that small errors in data can create large deviations in solution.

We have a mathematically precise notion of "strength." Let's now turn to the possible worlds, i.e., arrangements of facts, that we wish to consider. So as to make the problem mathematically tractable, we need to characterize these worlds with equations. The more general we are here the better. Because so many physical worlds are described by equations in physics are of this form, let us concentrate on second-order linear partial differential equations in $\mathbf{R}^{d}$:

$$[\sum_{i=1}^{d}\sum_{j=1}^{d} a_{ij}\frac{\partial}{\partial x_i}\frac{\partial}{\partial x_j}+\sum_{i=1}^{d} b_i\frac{\partial}{\partial x_i}+c]u=0 \qquad (1)$$

Here a_{ij} is a matrix, b_i a vector, and c a scalar; all three are differentiable functions of the coordinates. a_{ij} is sometimes called a "coefficient matrix" and it can be assumed to be symmetric without loss of generality. Scores of the most important equations in physics fit the form of (1), e.g., the wave equation, the heat equation, the Schrödinger equation, the Klein-Gordon equation, the Euler equation, the Poisson equation, parts of the Dirac equation. Many equations not in form (1) can be approximated by something in that form.

We can now state *Proposal 2*:

> For worlds described by (1), a temporal direction at point p of (M^{d},g) is that direction normal to the $(d$-1)-dimensional hypersurface intersecting p upon which Cauchy data can be prescribed to obtain a well-posed Cauchy problem.

For this claim to be plausible the temporal directions picked out by Proposal 2 had better mesh well with the directions physics normally picks out as temporal, and ideally, we could connect these directions to other features we normally attribute to time.

The idea behind the claim is that there is a sense in which well-posed Cauchy problems pick out temporal directions. Put very generally, a partial differential equation determines various invariant structures, e.g., a field of Monge cones, and these structures determine which hypersurfaces one can put data on to obtain well-posed Cauchy problems. What I suggest is turning this fact around and letting it implicitly define the timelike direction.

That is, imagine we have a manifold scattered with events. The events are compactly describable according to an equation of form (1). But we're blind as to which end of the manifold is "up", i.e., we're not told how to carve this into spacelike hypersurfaces evolving in the timelike directions. Instead we're told to seek informativeness, and in particular, a well-posed Cauchy problem. That, Proposal 2 asserts, will tell us what is timelike and spacelike. Will we find anything? And if we do, will that splitting correspond to what we normally consider to be time and space?

It is a remarkable feature of partial differential equations that we will find well-posed Cauchy problems and that the splitting required does correspond to what we ordinarily regard as the temporal and spatial directions. Indeed, that the Cauchy surface corresponds to what we ordinarily deem spatial is *necessary* for a well-posed Cauchy problem. The proof of this fact can't be described in detail here (see [5],[6],[7]); however, the argument, based on [6; 754-760], is described and put to similar but slightly different effect in [8].

Which equations of form (1) admit well-posed Cauchy problems? The answer is that only *hyperbolic* partial differential equations do (so long as we place our Cauchy data on non-closed hypersurfaces, which seems reasonable). It is a theorem that all linear hyperbolic second order systems have well-posed Cauchy problems, given certain mild assumptions. But it is also a mathematical fact that elliptic, parabolic and ultrahyperbolic equations of form (1) defined over a non-closed domain are *not* well-posed for Cauchy data. There is no single reason for this fact. Elliptic equations suffer a variety of fates: non-unique solutions, lack of existence, and lack of continuity. Parabolic equations have too many solutions given Cauchy data. And ultrahyperbolic equations are ruled out as an indirect consequence of Asgeirsson's 'mean-value' theorem [9].

For a concrete example, consider the hyperbolic Klein-Gordon equation

$$\frac{\partial^2\phi}{\partial t^2} = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} - m^2\phi \,.$$

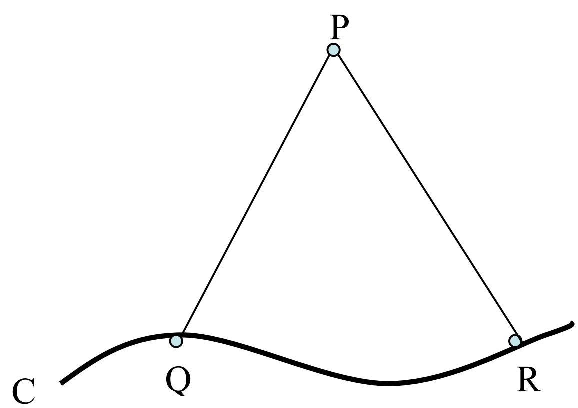
This equation, as is well-known, has a well-posed formulation. However, if we change the sign in the Klein-Gordon equation and let it go “LaPlacian”, the equation goes from being well-posed to ill-posed. Same goes if we allow the Klein-Gordon equation to turn into a diffusion equation.

Restricting attention to the hyperbolic case, it turns out that hyperbolic examples of (1) are extremely *finicky* when it comes to choice of "initial" Cauchy surface. Not just any choice of surface will do. The surfaces have to be, roughly, spatial.

Without getting mired in the details, let's see how this works in a toy example [10]. Consider the Cauchy problem for the one-dimensional nonhomogeneous wave equation,

$$u_{tt} - u_{xx} = f(x,t).$$

Normally we might specify the Cauchy data on a t=0 slice of the x-axis. However, we are interested in how the Cauchy problem distinguishes one direction as temporal; so rather than assuming the "initial" data fall on the x-axis, let us prescribe data on a arbitrary curve *C*. Take a point *P* and a curve *C* not containing *P*. If we have a solution of our equation at *P*, *u*(P), then there will be characteristic curves intersecting *P* and *C*, at different points on *C*, *Q* and *R*. The solution *u*(*P*), if it exists, will be determined by the values of u and u_x (where x is the locally normal direction) on the arc *QR* of *C* and the values within the "triangle" created by *QP*, *PR*, *RQ*.



Let us now define what it is to be "spacelike". *A curve C is spacelike if as P tends to a point on C, the points Q and R also tend to that same point on C.* Notice that this definition doesn't presume a prior time/space split. Yet it turns out that *u*(P) is consistent with the Cauchy data on *C only if C* is spacelike in this sense. In this way we see how strength can "find" the spatial and temporal directions. It would then remain to be shown that this distinguished timelike direction coalesces with what we know about time—which would be silly in this toy two-dimensional example.

Generalizations of the above ideas hold for all second-order hyperbolic equations. For hyperbolic equations, the characteristics are those level surfaces $\phi(x_0, x_1 \ldots x_n)$=constant whose normals to some hyperplane *C*, $\phi_{x0}, \phi_{x1} \ldots \phi_{xn}$, satisfy the first-order equation

$$\phi_{x_0}^2 = \sum_{j,k=1}^{n} a_{jk} \phi_{x_j} \phi_{x_k} \qquad (2)$$

for some x_0. If for variable x_0 and hyperplane C the normals to C satisfy the inequality

$$\phi_{x_0}^2 > \sum_{j,k=1}^{n} a_{jk}\phi_{x_j}\phi_{x_k} \tag{3}$$

then C is spacelike. In this way one sees the characteristic as a kind of limiting case of spacelike surfaces, where the inequality of (3) goes to the equality of (2).

Does this implicit definition of the spacelike/timelike correspond with that used in physics? It's no surprise that the answer is 'yes'. The main properties of hyperbolic equations are found in the wave equation, yet the Lorentz transformations that define relativistic physics are the group of linear transformations that do not alter the form of the wave equation. The sense of timelike defined in partial differential equation textbooks is a generalization and extension of the sense of timelike given in relativity.

Not only does the direction of strength pick out the temporal directions, but it also welds together many otherwise disparate features of time. In particular, note that it joins the three features of time mentioned in section 1:

> A. For *any* partial differential equation, the submanifold upon which ones places initial data must be *d*-1 dimensional (where *M* is *d*-dimensional) *if* one wants a well-posed Cauchy problem.
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> B. The *signature* of (M,g) is connected to the *type* of fundamental equations. The fundamental differential equations (in the coordinate representation) of a theory indicate the signature of spacetime. Indeed, for covariant field equations the matrix a_{ij} in (1) will have the same eigenvalues as the metric tensor. Not all equations used in physics will reflect the signature, of course, but the fundamental particle or field equations will.
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> C. In spacetimes with closed timelike curves, strictly speaking, there is no well-posed Cauchy problem because there are no Cauchy surfaces. A subset is a Cauchy surface if every inextendible worldine of the spacetime intersects the surface exactly once.

Fact A tells us that if we define time via Cauchy problems, it is bound to be one-dimensional. Fact B informs us that the signature will be Lorentzian. And Fact C warns us not to expect closed timelike curves in spacetimes with well-posed Cauchy problems.[5]

In sum, in the arena of worlds described by (1) and limited to desire for information via well-posed Cauchy problems, one can rigorously demonstrate that strength nicely divides time from space; in addition, one can connect the three features we initially associated with time: the ways in which it is special topologically, metrically and with respect to free mobility.

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[5] Less strictly, one can define "generalized Cauchy surfaces" a la [4] and this association fails; one still expects something in the spirit of Section 3 to hold, however, for CTC's will typically impose restrictions on initial data.

5. CAVEATS AND CONJECTURES

The real world is far messier than the model of Section 4. Having a well-posed Cauchy problem is neither necessary nor sufficient for a theory being informative. Theories can be more or less informative in a variety of ways. Moreover, there are many theories whose central equations are not of form (1). For these reasons and others I understand the result of Section 4 as merely a kind of ideal Platonic illustration of the more general claim of Section 3.

That said, I expect many of the associations proven in Section 4 to hold in varying degrees even when we go outside the scope of that problem. First, many equations not of form (1) can be approximated by those of form (1). By choosing small neighborhoods we can approximate nonlinear equations, by introducing new auxiliary fields we can get higher-order equations, and so on. Second, for quasi-linear symmetric first-order equations, which are capable of representing virtually every system of physical interest, one can make plausible an existence and uniqueness claim. More importantly for us, one of the conditions for this is what Geroch [11] calls a 'hyperbolization', and this demands a spacelike versus timelike distinction once again. So results not as strong as that above but still in the neighborhood may be possible.

In the end, it's a remarkable fact that the fundamental laws of nature – be they Newtonian, quantum or relativistic – possess an overwhelming asymmetry in informativeness. Newtonian mechanics and quantum mechanics are not even remotely "deterministic" in the directions we call spatial. In the smattering of results we have in general relativity, there are no known "sideways" Cauchy problems [12]; and if we can get them – and here is a conjecture – they will be far less simple than the timelike Cauchy problems. I submit that this previously unmentioned asymmetry between time and space is in fact responsible for why we think the temporal directions are special.

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